Type System for Egison

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Syntax

c denotes a character.
s denotes a string.
n denotes a number.
b denotes a boolian.
C denotes a constructor of an inductive data.
C_p denotes a constructor of an inductive pattern.
C_{dp} denotes a constructor of an inductive data pattern.

Expressions		
M, N	$::= x \mid c \mid s \mid b \mid n$	variables and constants
	$\mid$ $\texttt{(if } M\ M\ M\texttt{)}$	
	$\mid$ $\texttt{(lambda [\$}x\ \texttt{\$}y\ ...\texttt{]}\ M\texttt{)}$	abstraction
	$\mid$ $\texttt{(}M\ M\ ...\texttt{)}$	application
	$\mid$ $\texttt{[}M\ ...\texttt{]}$	tuple
	$\mid$ $\{M\ ...\}$	collection
	$\mid$ $\langle C\ M\ ...\rangle$	inductive data
	$\mid$ $\texttt{something}$	something matcher
	$\mid$ $\texttt{(match-all }M\ M\ \texttt{[}p\ M\texttt{])}$	
	$\mid$ $\texttt{(matcher }\{\phi\ ...\}\texttt{)}$	
Patterns		
p	$::= \_$	wildcard
	$\mid$ $\texttt{\$}x$	pattern variable
	$\mid$ $\texttt{,}M$	value pattern
	$\mid$ $\langle C_p\ p\ ...\rangle$	inductive pattern
Matcher Clauses		
ϕ	$::= \texttt{[}pp\ M\ \{\texttt{[}dp\ M\texttt{]}\}\texttt{]}$	
Primitive Pattern Patterns		
pp	$::= \texttt{\$}$	pattern hole
	$\mid$ $\texttt{,\$}x$	value-pattern pattern
	$\mid$ $\langle C_{pp}\ pp\ ...\rangle$	inductive-pattern pattern
Primitive Data Patterns		
dp	$::= \texttt{\$}x$	primitive pattern variable
	$\mid$ $\langle C_{dp}\ dp\ ...\rangle$	inductive-data pattern
Types		
S, T	$::= X \mid I \mid \texttt{String} \mid \texttt{Integer} \mid \texttt{Bool}$	
	$\mid$ $T \to T$	
	$\mid$ $\texttt{[}T\ T\ ...\texttt{]}$	
	$\mid$ $\{T\}$	
	$\mid$ $\texttt{(Pattern }T\texttt{)}$	
	$\mid$ $\texttt{(PPPattern }T\ T\texttt{)}$	
	$\mid$ $\texttt{(PDPattern }T\texttt{)}$	
	$\mid$ $\texttt{(Matcher }T\texttt{)}$	
Contexts		
Γ	$::= \epsilon$	empty context
	$\mid$ $\Gamma, x : T$	append x to the context
	$\mid$ $\Gamma +\!\!+\ \Gamma$	join two contexts

Typing Rules

$$\frac{}{\Gamma, x : T, \Delta \vdash x : T}\text{T-Var} \qquad \frac{}{\Gamma \vdash s : \texttt{String}}\text{T-Str} \qquad \frac{}{\Gamma \vdash n : \texttt{Integer}}\text{T-Num} \qquad \frac{}{\Gamma \vdash b : \texttt{Bool}}\text{T-Bool}$$

$$\frac{\Gamma \vdash M_1 : \texttt{Bool} \qquad \Gamma \vdash M_2 : T \qquad \Gamma \vdash M_3 : T}{\Gamma \vdash \texttt{(if } M_1\ M_2\ M_3\texttt{)} : T}\text{T-If}$$

$$\frac{\Gamma +\!\!+ \{x_1 : S_1, x_2 : S_2, ...x_n : S_n\} \vdash M : T}{\Gamma \vdash \texttt{(lambda [\$}x_1\ \texttt{\$}x_2\ ...\ \texttt{\$}x_n\texttt{]}\ M\texttt{)} : \texttt{[}S_1, S_2, ...\texttt{]} \to T}\text{T-Abs}$$

$$\frac{\Gamma \vdash M : \texttt{[}S_1\ S_2\ \cdots S_n\texttt{]} \to T \qquad \Gamma \vdash N_1 : S_1 \qquad \Gamma \vdash S_2 : T_2 \quad \cdots \quad \Gamma \vdash N_n : S_n}{\Gamma \vdash \texttt{(}M\ N_1\ N_2\ \cdots\ N_n\texttt{)} : T}\text{T-App}$$

$$\frac{\Gamma \vdash M_1 : T_1 \qquad \Gamma \vdash M_2 : T_2\ \cdots\ \Gamma \vdash M_n : T_n}{\Gamma \vdash \texttt{[}M_1\ M_2\ ...\ M_n\texttt{]} : \texttt{[}T_1\ T_2\ ...\ T_n\texttt{]}}\text{T-Tuple}$$

$$\frac{\Gamma \vdash M_1 : T \qquad \Gamma \vdash M_2 : T\ ...\ \Gamma \vdash M_n : T}{\Gamma \vdash \{M_1\ M_2\ ...\ M_n\} : \{T\}}\text{T-Collection}$$

$$\frac{\Gamma \vdash C : \texttt{[}S_1\ S_2\ ...S_n\texttt{]} \to T \qquad \Gamma \vdash N_1 : S_1 \qquad \Gamma \vdash N_2 : S_2\ ...\ \Gamma \vdash N_n : S_n}{\Gamma \vdash \langle C\ N_1\ N_2\ ...\ N_n\rangle : T}\text{T-InductiveData}$$

$$\frac{\Gamma \vdash M_1 : T_1 \qquad \Gamma \vdash M_2 : \texttt{(Matcher } T_1\texttt{)} \qquad \Gamma \vdash p : \texttt{(Pattern } T_1\texttt{)} \qquad \Gamma +\!\!+ V(\Gamma, p) \vdash M_3 : T_3}{\Gamma \vdash \texttt{(match-all } M_1\ M_2\ \texttt{[}p\ M_3\texttt{])} : \{T_3\}}\text{T-MatchAll}$$

$$\frac{}{\Gamma \vdash \texttt{something} : \texttt{(Matcher } T\texttt{)}}\text{T-Something}$$

$$\frac{\begin{array}{c}\Gamma \vdash pp_i : \texttt{(PPPattern } T\ \texttt{[}S_k\texttt{]}_k\texttt{)} \qquad \Gamma \vdash M_i : \texttt{(Matcher [}S_k\texttt{]}_k\texttt{)} \\ \Gamma \vdash dp_{ij} : \texttt{(PDPattern } T\texttt{)} \qquad \Gamma +\!\!+ V_{PP}(\Gamma, pp_i) +\!\!+ V_{DP}(\Gamma, dp_i) \vdash N_{ij} : \{\texttt{[}S_k\texttt{]}_k\} \qquad (\forall i, j)\end{array}}{\Gamma \vdash \texttt{(matcher [}pp_i\ M_i\ \texttt{[}dp_{ij}\ N_{ij}\texttt{]}_j\texttt{]}_i\texttt{)} : \texttt{(Matcher } T\texttt{)}}\text{T-Matcher}$$

Pattern:

$$\frac{}{\Gamma \vdash \texttt{\_} : \texttt{(Pattern } T\texttt{)}}\text{T-Wildcard}$$

$$\frac{}{\Gamma \vdash \texttt{\$}x : \texttt{(Pattern } T\texttt{)}}\text{T-PatternVariable}$$

$$\frac{\Gamma \vdash M : T}{\Gamma \vdash \texttt{,}M : \texttt{(Pattern } T\texttt{)}}\text{T-ValuePattern}$$

$$\frac{\begin{array}{c}\Gamma \vdash C_p : \texttt{[(Pattern } S_1\texttt{) (Pattern } S_2\texttt{)}\ \cdots\ \texttt{(Pattern } S_n\texttt{)]} \to \texttt{(Pattern } T\texttt{)} \\ \Gamma \vdash M_1 : \texttt{(Pattern } S_1\texttt{)} \qquad \Gamma +\!\!+ V(\Gamma, M_1) \vdash M_2 : \texttt{(Pattern } S_2\texttt{)} \\ \Gamma +\!\!+ V(\Gamma, M_1) +\!\!+ V((\Gamma +\!\!+ V(\Gamma, M_1)), M_2) \vdash M_3 : \texttt{(Pattern } S_3\texttt{)} \\ \cdots \quad \Gamma +\!\!+ V(\Gamma, M_1) +\!\!+\ \cdots +\!\!+ V(\Gamma +\!\!+ V(\Gamma, M_1) +\!\!+\ \cdots, M_{n-1}) \vdash M_n : \texttt{(Pattern } S_n\texttt{)}\end{array}}{\Gamma \vdash \langle C_p\ M_1\ M_2\ ...\ M_n\rangle : \texttt{(Pattern } T\texttt{)}}\text{T-InductivePattern}$$

Primitive pattern pattern:

$$\frac{}{\Gamma \vdash \texttt{\$} : \texttt{(PPPattern } T\ \texttt{[}T\texttt{])}}\text{T-PatternHole}$$

$$\frac{}{\Gamma \vdash \texttt{,\$}x : \texttt{(PPPattern } T\ \texttt{[])}}\text{T-ValuePatternPattern}$$

$$\frac{\Gamma \vdash C_{pp} : \texttt{[}T_1\ T_2\ ...T_n\texttt{]} \to T \qquad \Gamma \vdash pp_i : \texttt{(PPPattern } T_i\ S_i\texttt{)}}{\Gamma \vdash \langle C_{pp}\ pp_i\rangle_i : \texttt{(PPPattern } T\ \Sigma_i S_i\texttt{)}}\text{T-InductivePatternPattern}$$

Primitive data pattern:

$$\frac{}{\Gamma \vdash \texttt{\$}x : \texttt{(PDPattern } T\texttt{)}}\text{T-PrimitivePatternVariable}$$

$$\frac{\Gamma \vdash C_{dp} : \texttt{[}S_1\ S_2\ ...S_n\texttt{]} \to T \qquad \Gamma \vdash dp_1 : S_1 \qquad \Gamma \vdash dp_2 : S_2\ ...\ \Gamma \vdash dp_n : S_n}{\Gamma \vdash \langle C_{dp}\ dp_1\ dp_2\ ...\ dp_n\rangle : T}\text{T-InductiveDataPattern}$$

V, V_{PP}, and V_{DP} return the type bindings for the variables in a pattern, primitive-pattern-pattern, and primitive-data-pattern, respectively.

$$
\begin{aligned}
V(\Gamma, \_) &= \epsilon \\
V(\Gamma, \$x) &= \{(x : T)\} && \text{when } \Gamma \vdash \$x : (\texttt{Pattern}\ T) \\
V(\Gamma, \texttt{,}M) &= \epsilon \\
V(\Gamma, \langle C\ p_1\ p_2\ \cdots\rangle) &= V(\Gamma, p_1) +\!\!+\ V(\Gamma +\!\!+\ V(\Gamma, p_1), p_2) +\!\!+\ \cdots
\end{aligned}
$$

$$
\begin{aligned}
V_{PP}(\Gamma, \$) &= \epsilon \\
V_{PP}(\Gamma, \texttt{,}\$x) &= \{(x : (\texttt{Pattern}\ T))\} && \text{when } \Gamma \vdash \$x : (\texttt{PPPattern}\ T) \\
V_{PP}(\Gamma, \langle C\ pp_1\ pp_2\ \cdots\rangle) &= V_{PP}(\Gamma, pp_1) +\!\!+\ V_{PP}(\Gamma, pp_2) +\!\!+\ \cdots
\end{aligned}
$$

$$
\begin{aligned}
V_{DP}(\Gamma, \$x) &= \{(x : T)\} && \text{when } \Gamma \vdash \$x : (\texttt{PDPattern}\ T) \\
V_{DP}(\Gamma, \langle C\ dp_1\ dp_2\ \cdots\rangle) &= V_{DP}(\Gamma, dp_1) +\!\!+\ V_{DP}(\Gamma, dp_2) +\!\!+\ \cdots
\end{aligned}
$$

Q&A

- Does Egison type system support type polymorphism?
  Let polymorphism has been implemented in our type system. But it is omitted to simplify this handout.

Examples

$$
\dfrac{}{\epsilon \vdash \texttt{"Hello"} : \texttt{String}}\ \text{T-Str} \qquad \dfrac{}{\{x : \texttt{Integer}\} \vdash \texttt{"World!"} : \texttt{String}}\ \text{T-Str}
$$

$$
\dfrac{\dfrac{}{\epsilon \vdash \texttt{true} : \texttt{Bool}}\ \text{T-Bool} \quad \dfrac{}{\epsilon \vdash \texttt{10} : \texttt{Integer}}\ \text{T-Num} \quad \dfrac{}{\epsilon \vdash \texttt{20} : \texttt{Integer}}\ \text{T-Num}}{\epsilon \vdash \texttt{(if true 10 20)} : \texttt{Integer}}\ \text{T-If}
$$

$$
\dfrac{\dfrac{\dfrac{}{\cdots \vdash \texttt{b.+} : \texttt{[Integer Integer]} \rightarrow \texttt{Integer}}\ \text{T-Var} \quad \dfrac{}{\cdots \vdash x : \texttt{Integer}}\ \text{T-Var} \quad \dfrac{}{\cdots \vdash \texttt{10} : \texttt{Integer}}\ \text{T-Num}}{\{\texttt{b.+} : \texttt{[Integer Integer]} \rightarrow \texttt{Integer}\} +\!\!+\ \{x : \texttt{Integer}\} \vdash \texttt{(b.+ x 10)} : \texttt{Integer}}\ \text{T-App}}{\{\texttt{b.+} : \texttt{[Integer Integer]} \rightarrow \texttt{Integer}\} \vdash \texttt{(lambda [\$x] (b.+ x 10))} : \texttt{Integer} \rightarrow \texttt{Integer}}\ \text{T-Abs}
$$